

AS Level Mathematics B (MEI)
H630/01 Pure Mathematics and
Mechanics

Question Set 1

1. Write $\frac{8}{3-\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers to be found. (2)

$$\frac{8}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{24+8\sqrt{5}}{4} = \underline{6+2\sqrt{5}} \Rightarrow a+b\sqrt{5} \text{ with } \underline{a=6} \text{ and } \underline{b=2}$$

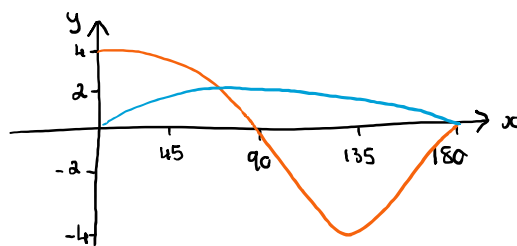
$$(3-\sqrt{5})(3+\sqrt{5}) = 9-5$$

2. Find the binomial expansion of $(3 - 2x)^3$. (4)

$$(3-2x)^3 = \binom{3}{0}(3)^0(-2x)^3 + \binom{3}{1}(3)^1(-2x)^2 + \binom{3}{2}(3)^2(-2x)^1 + \binom{3}{3}(3)^3(-2x)^0$$

$$= \underline{-8x^3 + 36x^2 - 54x + 27}$$

3.
(i) Sketch the graphs of $y = 4\cos(x)$ and $y = 2\sin(x)$ for $0^\circ \leq x \leq 180^\circ$ on the same axes.



$$y = 4\cos x \quad (2)$$

$$y = 2\sin x$$

Note we only show $0^\circ \leq x \leq 180^\circ$

- (ii) Find the exact coordinates of the point of intersection of these graphs, giving your answer in the form $(\arctan(a), k\sqrt{b})$, where a and b are integers and k is rational. (4)

$$4\cos x = 2\sin x$$

$$1 = \frac{2\sin x}{4\cos x} = \frac{1}{2}\tan x \Rightarrow \tan x = 2 \Rightarrow x = \arctan(2) = \arctan(a) \text{ with } a = 2.$$

$$\text{Then } y = 4\cos(\arctan(2)) = \frac{4\sqrt{5}}{5} \Rightarrow \text{Point of intersection is } (\arctan(a), k\sqrt{b})$$

$$\text{where } a = 2, k = \frac{4}{5} \text{ and } \underline{b=5}.$$

- (iii) A student argues that without the condition $0^\circ \leq x \leq 180^\circ$ all the points of intersection of the graphs would occur in the intervals of 360° because both $\sin(x)$ and $\cos(x)$ are periodic functions with this period. Comment on the validity of the student's argument. (1)

This is not valid, as there is a point of intersection every 180° .

In this question you must show detailed reasoning.

4. You are given that $f(x) = 4x^3 - 3x + 1$.

- (i) Use the factor theorem to show that $(x + 1)$ is a factor of $f(x)$. (2)

Factor Theorem: $f(-1) = 0$

$$\Rightarrow f(-1) = 4(-1)^3 - 3(-1) + 1 = 0 \text{ as required } \Rightarrow \underline{x+1} \text{ is a factor.}$$

- (ii) Solve the equation $f(x) = 0$. (3)

Long Division:

$$\begin{array}{r}
 - 4x + 1 \\
 x+1 \overline{) 4x^3 - 3x + 1} \\
 \underline{-4x^3 - 4x^2} \\
 -4x^2 - 3x + 1 \\
 \underline{4x^2 + 4x} \\
 x + 1 \\
 \underline{-x - 1} \\
 0
 \end{array}$$

$$\Rightarrow f(x) = (x+1)(4x^2 - 4x + 1)$$

$$f(x) = (x+1)(2x-1)(2x-1) \Rightarrow \underline{x = -1} \text{ and } \underline{x = \frac{1}{2}}$$

$$\frac{4x^3}{x} = 4x^2 \Rightarrow 4x^2(x+1) = 4x^3 + 4x^2$$

$$\frac{-4x^2}{x} = -4x$$

$$-4x(x+1)$$

$$-4x^2 - 4x$$

$$\frac{x}{x} = 1 \Rightarrow x+1$$

$$\left. \begin{array}{l} 4x^2 - 4x + 1 \\ (2x-1)(2x-1) \end{array} \right\}$$

9.52

↓

10.46

54 mins

In this question you must show detailed reasoning.

5. Fig. 5 shows the graph of a quadratic function. The graph crosses the axes at the points $(-1, 0)$, $(0, -4)$ and $(2, 0)$.

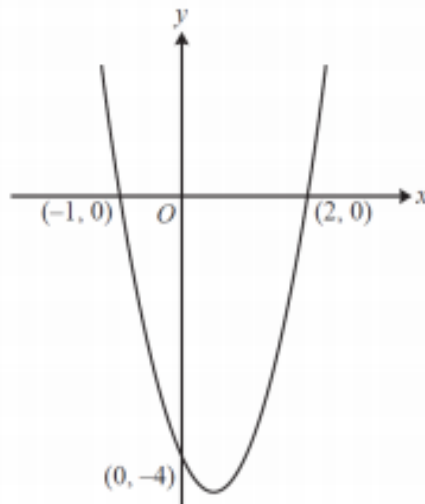


Fig. 5

Find the area of the finite region bounded by the curve and the x -axis. (8)

We can find the area of the finite region bounded by the curve and the x -axis by integrating.

We must first find the equation of our parabola and work out our limits before we can integrate.

We know the parabola will have equation with form $y = ax^2 + bx + c$.

We can see that $c = -4$ as this will be the y -intercept.

We have x -roots such that $x = -1$ and $x = 2 \Rightarrow f(x) = (x+1)(x-2)$

$$= x^2 - 2x + x - 2$$
$$= x^2 - x - 2 \quad \text{but we know that } c = -4, \text{ and we can achieve this by multiplying by 2.}$$

$\Rightarrow y = \underline{2x^2 - 2x - 4}$

Then our limits will be -1 and 2 . $\frac{2x^2}{2} = x^2$

$$\Rightarrow A = \int_{-1}^2 2x^2 - 2x - 4 \, dx = \left[\frac{2x^3}{3} - x^2 - 4x \right]_{-1}^2$$

$= -\frac{20}{3} - \frac{7}{3} = -9$ \leftarrow since below x -axis

$\Rightarrow \underline{A = 9}$

Total Marks for Question Set 1: 26

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