OCR
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Model Solutions

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Question Set 1

1. Write $\frac{8}{3-\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers to be found. **(2)**

$$\frac{8}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{24+8\sqrt{5}}{4} = \underline{\underline{6+2\sqrt{5}}} \Rightarrow a+b\sqrt{5} \text{ with } \underline{\underline{a=6}} \text{ and } \underline{\underline{b=2}}$$

$$(3-\sqrt{5})(3+\sqrt{5}) = 9-5$$

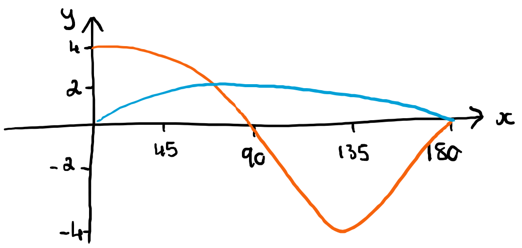
2. Find the binomial expansion of $(3-2x)^3$. **(4)**

$$(3-2x)^3 = \binom{3}{0}(3)^0(-2x)^3 + \binom{3}{1}(3)^1(-2x)^2 + \binom{3}{2}(3)^2(-2x)^1 + \binom{3}{3}(3)^3(-2x)^0$$

$$= \underline{\underline{-8x^3 + 36x^2 - 54x + 27}}$$

3.

(i) Sketch the graphs of $y = 4\cos(x)$ and $y = 2\sin(x)$ for $0° \leq x \leq 180°$ on the same axes. **(2)**



$y = 4\cos x$
$y = 2\sin x$

Note we only show $0° \leq x \leq 180°$

(ii) Find the exact coordinates of the point of intersection of these graphs, giving your answer in the form $(\arctan(a), k\sqrt{b})$, where a and b are integers and k is rational. **(4)**

$4\cos x = 2\sin x$

$1 = \frac{2\sin x}{4\cos x} = \frac{1}{2}\tan x \Rightarrow \tan x = 2 \Rightarrow x = \arctan(2) = \arctan(a)$ with $a = 2$.

Then $y = 4\cos(\arctan(2)) = \frac{4\sqrt{5}}{5} \Rightarrow$ Point of intersection is $(\arctan(a), k\sqrt{b})$

where $a = 2$, $k = \frac{4}{5}$ and $\underline{\underline{b=5}}$.

(iii) A student argues that without the condition $0° \leq x \leq 180°$ all the points of intersection of the graphs would occur in the intervals of $360°$ because both $\sin(x)$ and $\cos(x)$ are periodic functions with this period. Comment on the validity of the student's argument. **(1)**

This is not valid, as there is a point of intersection every $\underline{180°}$.

In this question you must show detailed reasoning.

4. You are given that $f(x) = 4x^3 - 3x + 1$.

(i) Use the factor theorem to show that $(x + 1)$ is a factor of f(x). **(2)**

Factor Theorem: $f(-1) = 0$

$\Rightarrow f(-1) = 4(-1)^3 - 3(-1) + 1 = 0$ as required $\Rightarrow$ $\underline{x+1}$ is a factor.

(ii) Solve the equation $f(x) = 0$. **(3)**

Long Division:

$$\begin{array}{r|l} & 4x^2 - 4x + 1 \\ \hline x+1 & 4x^3 - 3x + 1 \\ & -4x^3 - 4x^2 \\ \hline & -4x^2 - 3x + 1 \\ & 4x^2 + 4x \\ \hline & x + 1 \\ & -x - 1 \\ \hline & 0 \end{array}$$

$\frac{4x^3}{x} = 4x^2 \Rightarrow 4x^2(x+1)$
$= 4x^3 + 4x^2$

$\frac{-4x^2}{x} = -4x$

$-4x(x+1)$
$-4x^2 - 4x$

$\frac{x}{x} = 1 \Rightarrow x + 1$

$4x^2 - 4x + 1$
$\hookrightarrow (2x-1)(2x-1)$

9.52

10.46

54 mins

$\Rightarrow f(x) = (x+1)(4x^2 - 4x + 1)$

$f(x) = (x+1)(2x-1)(2x-1) \Rightarrow \underline{x = -1}$ and $\underline{x = \frac{1}{2}}$

In this question you must show detailed reasoning.

5. Fig. 5 shows the graph of a quadratic function. The graph crosses the axes at the points $(-1,0)$, $(0,-4)$ and $(2,0)$.

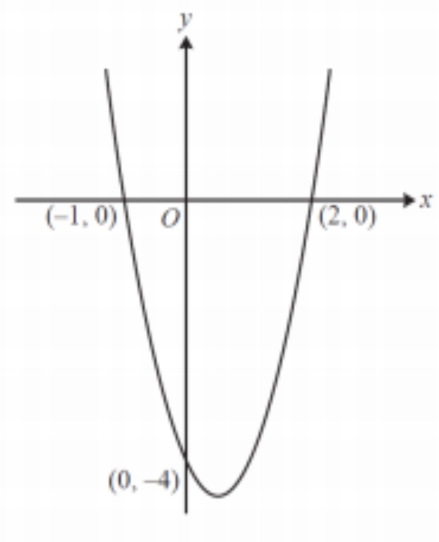


Fig. 5

Find the area of the finite region bounded by the curve and the x-axis. **(8)**

We can find the area of the finite region bounded by the curve and the x-axis by integrating.
We must first find the equation of our parabola and work out our limits before we can integrate.

We know the parabola will have equation with form $y = ax^2 + bx + c$.

We can see that $c = -4$ as this will be the y-intercept.

We have x-roots such that $x = -1$ and $x = 2 \Rightarrow f(x) = (x+1)(x-2)$

$$= x^2 - 2x + x - 2$$

$$= x^2 - x - 2$$

but we know that $c = -4$, and we can achieve this by multiplying by 2.

$$\Rightarrow y = 2x^2 - 2x - 4$$

Then our limits will be -1 and 2.

$\frac{2x^2}{2} = x^2$

$$\Rightarrow A = \int_{-1}^{2} 2x^2 - 2x - 4 \, dx = \left[\frac{2x^3}{3} - x^2 - 4x \right]_{-1}^{2}$$

< 0 since below x-axis

$$= \frac{-20}{3} - \frac{7}{3} = -9$$

$$\Rightarrow A = 9$$

Total Marks for Question Set 1: 26